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Simplified Rapid Switching Gain Scheduling for a Class of LPV Systems

Arvin Dehghani, Michael C. Rotkowitz, Brian D. O. Anderson, and Sung H. Cha

***Abstract*—A limitation of the original gain-scheduling approaches is that the closed-loop stability can only be assured when the underlying parameters vary sufficiently slowly. A remedy exists but requires for its implementation the possible solution of asymptotic Riccati equations (ARE) for an infinite number of different parameter values, and the on-line solution of a Riccati differential equation (RDE) with time-varying coefficient matrices. Our method avoids solving the RDE online and instead uses an explicit transient formula that looks up the predetermined solutions of the associated AREs at a finite set of given system operating points. Furthermore, only a finite number of AREs are solved to determine a finite set of controller gains.**

***Index Terms*—Asymptotic Riccati equations (ARE), Riccati differential equation (RDE.**

I. Introduction

Gain scheduling is a control method for nonlinear systems that utilizes a family of linear controllers, each of which provides satisfactory control for a different operating point of the system. The so-called scheduling variables, or indeed observable variables, determine the current operating region of the system and enable the appropriate choice of the linear controller. Gain scheduling can be viewed as an intuitive form of adaptive control, and allows the designer to choose any suitable control design method for implementing the local controllers [1], [2]. Once a set of local controllers is determined, a scheduling mechanism selects and switches a controller (or part of it) in the closed-loop while the plant and its operating points or associated physical parameters change over time. In fact, the idea of gain scheduling is well suited for linear parameter varying (LPV) systems, and nonlinear systems where the nonlinearities are due to physical parameter variation, see, e.g., [3]–[6]. In effect, the ideas of gain scheduling can also be viewed as a form of multiple model adaptive control (MMAC) [7] implementation. The work of this note pursues the possibility of exploiting a viable approach to gain scheduling in the MMAC context for the LPV systems [1], [2], [8].

For some time, it has been recognized that there is a theoretical gap in many of the formulations of gain scheduled controllers. The fact that switching occurs—even among controller-plant pairs which individually constitute stable closed-loops—raises a potential for instability. It is known that if the switching is very slow, instability cannot occur, but this sort of result is conservative; see, e.g., [9]. One way this potential instability difficulty is addressed involves results on LPV

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plants and LPV controllers with the parameter (which may vary arbitrarily fast) passed exactly from the plant to the controller. However, these results guarantee quadratic stability, which is an unnecessarily demanding concept. Apart from such conservatism, the contemporary theory has difficulty demonstrating that if there are continuing slow changes or rare rapid changes of the operating point, then stability is retained.

In developing supporting theories that guarantee the behavior of the scheduled controller in a process of repeated design syntheses associated with some scheduling strategy connecting locally designed controllers, an important step appears to be the formulation of the gain-scheduling design in the context of convex semidefinite programming expressed in terms of linear matrix inequalities (LMIs); see, e.g., [8], [10], [11]. Another vital step in the characterization of the aforementioned controller is the search for adequate Lyapunov functions. These functions are generally not readily obtainable. The use of a fixed Lyapunov function, as opposed to one that depends on the scheduled variables, resulted in the linear-fractional transformation (LFT) gain-scheduling techniques [12], [13] or the so-called quadratic gain-scheduled techniques [14]. Such approaches, however, are conservative since they allow for arbitrary rates of variation in the scheduled variables [15]. Even worse is that some systems are not quadratically stabilizable, i.e., are not stabilizable on the basis of a single Lyapunov function [15], [16]. Exploiting the ideas of parameter-dependent Lyapunov functions [15], which allow the incorporation of information on the rate of variation in the analysis or synthesis technique, can lead to a less conservative design. However, the implementations of such designs may require the real-time measurement of the parameter and even its time derivative, which is not available or hard to estimate. Attempts to overcome such restrictive implementation requirement, specially on the time derivative of the parameter, amount to utilizing a fixed Lyapunov function for the parameter-dependent control problem and hence compromise heavily on the performance [3]. Ensuring stability in such systems is also still an active area of research, e.g., [17].

A particular way of synthesizing controllers with potentially fast variations of parameters is suggested in [1], [18] but the method demands large computational power. The results in [18] suggest a particular way of synthesizing the gain scheduled controller. To ensure stability, the operating point is assumed to vary continuously and the plant is assumed to be minimum-phase at every operating point. A linear quadratic Gaussian (LQG) design is performed at each operating point using the ideas of loop transfer recovery (LTR) [19], [20] with the state feedback gain arranged to achieve very fast eigenvalues for its associated dynamics, and with the observer gain used to shape the effective overall closed-loop dynamics. The gain at any one operating point is obtained via the steady state solution of the algebraic Riccati equation (ARE) with matrices corresponding to the operating point. The observer gain on the other hand is determined using the solution of a time-varying Riccati differential equation (RDE). However, this approach is not computationally viable due to the requirement of solving the RDE and an infinite number of control AREs. We address these problems by proposing an approach which avoids solving the RDE online and instead uses an explicit transient formula that looks up the predetermined solutions of the associated AREs at a finite set of given system operating points and connects them in a standard way. Furthermore, only a finite number of AREs are solved to determine a finite set of controller gains. Our method offers a modification and computational simplification of the gain scheduling design of [18] but achieves similar level of performance.

Section II presents a short background leading to the introduction of the problems of interest. The main results are presented in Section III followed by the stability analysis of Section IV. The numerical example of Section V illustrates the efficacy of the proposed method. Section VI contains concluding remarks.

II. Background and Problem Setup

Consider a LPV system described by

$$\begin{aligned}\dot{x}(t) &= A(\Lambda(t))\,x(t) + B(\Lambda(t))\,u(t)\\ y(t) &= C(\Lambda(t))\,x(t)\end{aligned} \tag{1}$$

where $x(t)$, $u(t)$, and $y(t)$ are the state variable, (control) input, and system output, respectively. The time-varying system parameter $\Lambda : \mathbb{R}^+ \rightarrow \Lambda_{\rm box}$ can be measured online, where $\Lambda_{\rm box} := [-1, 1]^p$, $p \geq 0$. The matrix functions $A : \Lambda_{\rm box} \rightarrow \mathbb{R}^{n\times n}$, $B : \Lambda_{\rm box} \rightarrow \mathbb{R}^{n\times m}$ and $C : \Lambda_{\rm box} \rightarrow \mathbb{R}^{k\times n}$ are assumed to have bounded entries at all times. Here we suppose the system has $p \geq 0$ physical parameters and assume that the parameters remain bounded, and that the range of each parameter can be restricted to be symmetric about zero and within $[-1, 1]$ by variable substitutions and scaling. As in [18], the plant is assumed to be minimum-phase at every operating point. We also assume that the system in (1) is uniformly controllable and uniformly observable [21], and focus on LPV systems where their operating points or associated physical parameters—such as friction, mass and load, etc., are frequently changing over time. The scope could include special types of nonlinear systems, still addressable in the framework of LPV systems, where the nonlinearities are due to physical parameter variation.

A typical gain scheduling controller design procedure involves the following steps [2], [8], [22]. Essentially, different choices for each step will result in different gain scheduling methodologies at the end.

- Step 1: Identify a set of nominal operating points for the given system, and determine their characteristics—how often, how fast and how far their values are changing—with (physical) parameter variations;
- Step 2: Decide how to treat the local plants corresponding to the identified set of operating points via linearizations and/or reformulation;
- Step 3: Choose a design technique for a family of local linear controllers corresponding to the family of local LPV plant models;
- Step 4: Implement a scheduling algorithm to change the control gains for the family of local controllers;
- Step 5: Analyze and assess the overall performance and stability.

We deal with the problems in the Steps 3, 4, and 5 above, and assume that the assumptions in Steps 1 and 2 hold. The set of operating points and its characteristics (Step 1) are assumed to be available and the nonlinear plants are assumed to be linearized or reformulated to have a family of linear local models (Step 2). The Steps 3 and 4 will be addressed as a coupled problem using the idea of loop transfer recovery (LTR) with the time-varying Kalman filter.

The Step 1 is often classified as a general adaptive control problem and many involved aspects still remain for a methodological approach to be found [7]. The questions at this step are about how many operating points are adequate for achieving the desired performance and how one should place the identified operating points within the parameter space. At Step 2, two choices are linearizing a nonlinear system at each operating point resulting in a family of linearized local plant models and reformulating the original nonlinear equations as a global LPV model that hides its nonlinearities into the scheduling variable [18]. The third step, whether the given local LPV model is obtained from linearization or reformulation, involves the design of a family of local linear controllers corresponding to the family of local LPV plant models using any linear control design method. The fourth and fifth steps are often carried out together where the designer should take care of the instability and performance degradations during normal operations and switching between local controllers. Analytic and simulation-based verifications may be performed for local and global behavior of the designed controller. Next, two definitions are presented which are vital in presenting the results of the proceeding sections. In this manuscript, A' denotes the transpose of a matrix A.

Consider the time-varying system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t); \quad y(t) = C(t)x(t) \tag{2}$$

where $x(t_0)$ is given, and the matrices A, B and C are of compatible dimensions and with bounded entries.

Definition 1 (Uniform Observability [23]): The pair $(A(t), C(t))$ of the system in (2) is defined to be uniformly observable if and only if $\exists \delta_o, \alpha_o, \beta_o > 0$ such that

$$\alpha_o I \leq W(s, s+\delta_o) \leq \beta_o I, \quad \forall s \in \mathbb{R} \tag{3}$$

using the Observability Gramian $W : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^{n \times n}$

$$W(s,t) = \int_s^t \Phi'(\tau, s)C'(\tau)C(\tau)\Phi(\tau, s)d\tau \tag{4}$$

where Φ is the transition matrix corresponding to A.

Definition 2 (Uniform Controllability [23]): Uniform Controllability is defined as

$$\alpha_c I \leq M(s-\delta_c, s) \leq \beta_c I, \quad \forall s \in \mathbb{R} \tag{5}$$

for some $\delta_c, \alpha_c, \beta_c > 0$ in a dual manner to Definition 1 using the Controllability Gramian

$$M(s,t) = \int_s^t \Phi(t, \tau)B(\tau)B'(\tau)\Phi'(t, \tau)d\tau \tag{6}$$

where Φ is the transition matrix corresponding to A.

Note that if the system is uniformly controllable (or uniformly observable) it is also uniformly stabilisable (or uniformly detectable).

A. Computational Problems

A gain scheduling design involves designing several local controllers with the desired properties and performance at several operating points followed by a scheduling of the local controllers in an online manner. However, to preserve the desired properties and ensure performance and stability effectively, the parameter must be limited to vary slowly. This requirement fundamentally limits the applicability of the gain scheduling design. Given the assumption that for every parameter value the plant is minimum phase, the approach of [18] ensures a stable closed loop even with fast though bounded-rate parameter variation.

Let Γ, $Q(\kappa)$, and $R(\kappa) \forall \kappa \in \Lambda_{\text{box}}$ be symmetric matrix design parameters whose values are to be chosen; they must meet the LQR constraints of definiteness or semidefiniteness. For the LPV system in (1), the gain scheduling controller is a standard observer and state-feedback

$$\begin{aligned} \dot{\hat{x}} &= [A(\Lambda(t)) - B(\Lambda(t))G(\Lambda(t)) - H(t)C(\Lambda(t))] \times \hat{x} \\ &\quad - H(t)(r-y) \\ u &:= -G(\Lambda(t))\hat{x} \end{aligned} \tag{7}$$

where r, $\hat{x}$, u, and y are, respectively, the reference signal, state estimate, control, and system output. The observer gain $H : \mathbb{R}^+ \to \mathbb{R}^{n \times k}$

$$H(t) = \Sigma(t)C'(\Lambda(t))(R(\Lambda(t)))^{-1} \tag{8}$$

and the control feedback gain $G : \Lambda_{\text{box}} \to \mathbb{R}^{m \times n}$

$$G(\kappa) = \Gamma^{-1}B'(\kappa)Z(\kappa) \tag{9}$$

where $\Sigma(t)$ is the solution of the time-varying Kalman filter RDE

$$\begin{aligned} \dot{\Sigma}(t) &= \Sigma(t)A'(\Lambda(t)) + A(\Lambda(t))\Sigma(t) \\ &\quad + Q(\Lambda(t)) \\ &\quad - \Sigma(t)C'(\Lambda(t))(R(\Lambda(t)))^{-1}C(\Lambda(t))\Sigma(t) \end{aligned} \tag{10}$$

with $\Sigma(0) = \Sigma'(0) > 0$, and $Z(\kappa)$ is the stabilizing solution of the ARE

$$\begin{aligned} &Z(\kappa)A(\kappa) + A'(\kappa)Z(\kappa) + C'(\kappa)C(\kappa) \\ &\qquad - Z(\kappa)B(\kappa)\Gamma^{-1}B'(\kappa)Z(\kappa) = 0. \end{aligned} \tag{11}$$

Note that the control ARE in (11) is an infinite collection of steady state Riccati equations associated with a LTR approach to design, where the closed-loop state feedback dynamics are very fast. Here, since there exists a bound on the rate of variation of the physical parameter and the closed-loop feedback dynamics associated with

$$A_{\text{cont}}(\kappa) := A(\kappa) - B(\kappa)\Gamma^{-1}B'(\kappa)Z(\kappa) \tag{12}$$

are stable for each fixed κ, and have modes considerably faster than the rate of parameter variations, mean that the closed-loop dynamics associated with $A_{\text{cont}}(\Lambda(t))$ are also stable. This stability result follows from the fact that a system with time-varying parameters is stable if it is stable for all fixed values of the parameters and the parameters vary much more slowly than the system dynamics [24, Sec. IV.8]. Note also that it is standard in LTR design to ensure that either the dynamics associated with the state feedback law or those associated with the estimator are very fast; here, it is the dynamics associated with the state feedback law, and this ensures that the overall closed-loop dynamics obtained by using the controller based on feedback of a state estimate are mainly dominated by the observer dynamics, which become a form of target loop for the closed-loop dynamics. Of course, the LTR design requires that the original system be minimum phase or stably invertible.

In terms of the stability of the overall loop, the time-varying dynamics associated with the Kalman filter are stable under uniform controllability and uniform observability assumptions, with stability not being related to the speed of parameter variations [23]. The overall closed-loop becomes stable when both the state feedback dynamics and observer dynamics are stable. However, even if this gain scheduling design using loop transfer recovery works with fast parameter variation there is still a computational problem where the cost of solving the RDE online is concerned. A separate computational problem arises since an infinite number of control AREs are to be solved. Next, we will show how we can use a *finite* set of control AREs and express the observer RDE solution using a simple formula for the transient solution of a constant coefficient RDE, assuming its steady-state solution is known.

III. Proposed Control Synthesis

In this section, we shall first present an explicit formula that provides the transient solutions of a constant coefficient RDE given its steady state solution. Then, our proposed gain scheduling controller design is presented utilizing the filtering transient RDE solutions that converge to the predetermined stabilizing ARE solutions. This is aided by noting that the RDE has piecewise constant coefficients and exploiting the explicit formula.

A. Explicit Formula for Transient Solution of the RDE

A controller design is postulated below in which the RDE coefficients are piecewise constant and take one of a known finite set of values. Associated with each set of values, there is a steady-state solution of the RDE. We can also appeal to a result of [25], which proposes an explicit formula for the transient solution of the RDE (when $A(t)$, $C(t)$, $R(t)$, and $Q(t)$ are constant) computed *using knowledge of its value at the end of an interval and of the steady state solution*; i.e., the solution of the associated ARE. A dual filtering result below expresses the transient solution using the result above.

Lemma 3: Consider the time-invariant case of the linear system in (1) with constant matrices $\bar{A}$, $\bar{C}$, and constant design parameters $\bar{Q} = \bar{Q}' = \bar{L}\bar{L}' \geq 0$ and $\bar{R} = \bar{R}' > 0$, where $(\bar{A}, \bar{C})$ is detectable and $(\bar{A}, \bar{L})$ is stabilizable. Let $\bar{\Sigma} = \bar{\Sigma}' \geq 0$ be the stabilizing solution of the associated ARE, $\hat{A} := (\bar{A} - \bar{\Sigma}\bar{C}'\bar{R}^{-1}\bar{C})$ and let $\bar{W}$ satisfy

$$\bar{W}\hat{A}' + \hat{A}\bar{W} - \bar{C}\bar{R}^{-1}\bar{C}' = 0. \tag{13}$$

Then the transient solution of the RDE from any initial value $\Sigma(t_0) = J = J' \geq 0$ to the stabilizing solution $\bar{\Sigma}$ with $\tau := t - t_0, \tau \geq 0$ can be obtained via

$$\Sigma(\tau) = \bar{\Sigma} + \mathrm{e}^{\hat{A}\tau} \times \left[(J - \bar{\Sigma}) \left\{ I + \left(\bar{W} - \mathrm{e}^{\hat{A}'\tau}\bar{W}\mathrm{e}^{\hat{A}\tau} \right)(J - \bar{\Sigma}) \right\}^{-1} \right] \mathrm{e}^{\hat{A}'\tau}. \tag{14}$$

Proof: The proof is straightforward following the proof of the dual theorem in [25]. ■

Using Lemma 3, one can solve the RDE on an interval applicable to one operating point of the system by using a stored steady state solution value, the actual solution value immediately before the switching of operating point, and the transient formula (14).

B. Proposed Design Procedure

The gain scheduling controller for the LPV system of interest in (1) with the time-varying system parameter $\Lambda(t)$ requires the following assumptions and offline calculations:

a) Suppose a set of N operating points $\Lambda_{\text{oper}} := \{\Lambda_1, \Lambda_2, \ldots, \Lambda_N\} \subset \Lambda_{\text{box}}$ are identified *a priori* and that for each $\Lambda_i \in \Lambda_{\text{oper}}$ there is an associated open subset $\Xi_i \subset \Lambda_{\text{box}}$ such that $\Lambda_i \in \Xi_i$ and $\bigcup_{i=1}^{N} \Xi_i = \Lambda_{\text{box}}$. Let the set of all subsets be $\Xi_{\text{oper}} := \{\Xi_1, \Xi_2, \ldots, \Xi_N\}$.

b) Choose design parameters $\Gamma \in \mathbb{R}^{m\times m}, Q_i \in \mathbb{R}^{n\times n}, R_i \in \mathbb{R}^{k\times k}$ for each $i \in \{1, \ldots N\}$.

c) At each $\Lambda_i \in \Lambda_{\text{oper}}$, assume that the corresponding "frozen" form of the given system is a minimal realization with $A_i := A(\Lambda_i), B_i := B(\Lambda_i), C_i := C(\Lambda_i)$ such that (A_i, B_i) is controllable and (A_i, C_i) is observable $\forall i \in \{1, \ldots N\}$.

d) For each operating point $\Lambda_i \in \Lambda_{\text{oper}}$, determine a stabilizing solution $\bar{\Sigma}_i$ of the observer ARE for the corresponding values A_i, C_i, Q_i, R_i. Let the collection of all stabilizing solutions be $\bar{\Sigma}_{\text{oper}} := \{\bar{\Sigma}_1, \bar{\Sigma}_2, \ldots, \bar{\Sigma}_N\}$.

The proposed procedure below determines the feedback gain $G : \{1, \ldots N\} \to \mathbb{R}^{m\times k}$ and observer gain $H : \mathbb{R}^+ \to \mathbb{R}^{k\times k}$ for the observer/state feedback controller in (7), as well as a sequence of switching times $\{t_j\}_{j\in\mathbb{Z}_+}$ and an index function $c : \mathbb{R}^+ \to \{1, \ldots N\}$. Note that the feedback gain is completely determined by the index function as $G(\Lambda_{c(t)})$ via (9).

Procedure 4: The feedback gain G, observer gain H for the system in (7), a sequence of switching times $\{t_j\}_{j\in\mathbb{Z}_+}$, and an index function c are determined.

a) Initialize $j = 0, t_0 = 0$, and choose $c(t_0)$ such that $\Lambda(t_0) \in \Xi_{c(t_0)}$ and set $\Sigma(t_0) = \bar{\Sigma}_{c(t_0)}$.

b) Suppose at time t, $\Lambda(t)$ is not sampled again until not-necessarily known time t_+. Set $c(\tau) = c(t)\ \forall\tau \in (t, t_+)$, compute $\Sigma(\tau)\forall\tau \in [t, t_+)$ using the transient formula (14) with the initial value $J = \Sigma(t)$ and the steady state[1] value $\bar{\Sigma} = \bar{\Sigma}_{c(t)}$. Set the observer gain $\forall\tau \in [t, t_+)$ using

$$H(\tau) := \Sigma(\tau)C'_{c(t)}\left(R_{c(t)}\right)^{-1}. \tag{15}$$

c) Suppose we sample $\Lambda(t)$ at time t, and that we sampled last at t_-. If $\Lambda(t) \in \Xi_{c(t_-)}$, set $c(t) = c(t_-)$. Otherwise (i.e., $\Lambda(t) \notin \Xi_{c(t_-)}$), choose $c(t)$ such that $\Lambda(t) \in \Xi_{c(t)}$, set $t_{j+1} = t$, and increment $j = j + 1$.

d) Go to step ii. with next sampling time.

Note that at t the knowledge of the next sampling time t_+ is not required, meaning that c, and thus $\bar{\Sigma}$ and G, may remain fixed until the next switching time t_{j+1}. From Procedure 4

$$c(t) = c(t_j) \quad \forall t \in [t_j, t_{j+1}),\ \forall j \in \mathbb{Z}_+. \tag{16}$$

Here, we assume that we can sample Λ continuously, and hence Procedure 4 further yields

$$\Lambda(t) \in \Xi_{c(t)} \quad \forall t \in \mathbb{R}^+ \tag{17}$$

and

$$t_{j+1} = \inf\left\{t > t_j | \Lambda(t) \notin \Xi_{c(t_j)}\right\} \quad \forall j \in \mathbb{Z}^+. \tag{18}$$

Since the regions $\Xi_i \subset \Lambda_{\text{box}}$ are open and $A(t), B(t), C(t)$ are bounded $\forall t \in \mathbb{R}^+$, we have

$$\exists\delta_j > 0 \text{ such that } t_{j+1} - t_j \geq \delta_j \quad \forall j \in \mathbb{Z}^+.$$

We then define

$$\delta_c := \inf_{j\in\mathbb{Z}^+}(t_{j+1} - t_j). \tag{19}$$

From the uniform boundedness of A, B, C, the openness of $\Xi_i\forall i$, and the fact that N is finite, it follows that $\delta_c > 0$, facilitating the proof of uniform controllability in Lemma 6.

Remark 5: The solution of the RDE associated with a controller is normally computed backward in time whereas the solution of an RDE associated with an observer is computed forward in time. Thus, two different methods are used to obtain $H(t)$ and $G(\kappa)$.

In using the proposed design, there is no need to compute the solution of the actual RDE online. One only needs two simple online computations to determine the current subset $\Xi_i \in \Xi_{\text{oper}}$ for the measured system parameter $\Lambda(t)$ and schedule $\Sigma(t)$ via the pre-determined values $\bar{\Sigma}_i \in \bar{\Sigma}_{\text{oper}}$ and the transient formula of Lemma 3. In addition, the set of state feedback gains is pre-computed, and the appropriate gain is determined by the current subset Ξ_i.

IV. Stability Analysis of Proposed Gain Scheduling Design

Given the nature of the proposed design and for the pedagogical benefit, the stability of the proposed design will be verified in the following three regimes. First, the fixed-point case, where the plant and controller both assume a common constant value of Λ_i for $\Lambda(t)$, will be examined. Second, we consider the transition case when the plant parameter switches among a finite set Λ_{oper} and the controller is 'tuned' to the plant parameter. Last, we consider the parameter-varying case when the plant and controller do not assume in general the same value for $\Lambda(t)$ within the same subset Ξ_i at other than isolated times. The plant parameter is $\Lambda(t)$, and the controller parameter is Λ_i for some i, albeit

[1]Note that it would *not* be expected that the steady state value was attained. If $t_+ - t$ is sufficiently large, the steady state value is approached but in general it will not be exactly attained at time t_+.

with the time-varying observer gain depending on the past history of the set of $\Lambda_j \in \Lambda_{\rm oper}$ encountered. The final case corresponds to the procedure we have presented; its understanding is aided by considering the simpler cases first.

A. Fixed-point Analysis

Consider the system of interest in (1) and the proposed controller in (7) with a constant system parameter $\Lambda(t)$, say Λ_i, $\forall t$. The plant and controller will thus be time-invariant. The controller can be designed via normal LQG methods (albeit using LTR) ensuring the closed-loop stability. Instead of the observer gain $H(t)$ in (18) within the controller being determined via the solution of a RDE, it is a constant $H_i := (R_i)^{-1}\bar{\Sigma}_i C_i'$, $\bar{\Sigma}_i \in \bar{\Sigma}_{\rm oper}$.

B. Finite Parameter Set Analysis

Suppose that the underlying plant parameter value $\Lambda(t)$ switches among a finite set of values drawn from the set $\Lambda_{\rm oper}$, and that the state feedback gain incorporated in the controller switches instantaneously between corresponding values. Suppose also that the dynamics associated with state feedback are considerably faster than the average rate of parameter variation, so that we only need to focus on the stability of the observer dynamics. The combination of the observer and the state feedback defines the overall closed-loop system. We earlier described how the observer gain can be calculated using the known ARE solutions and the formula generating the Riccati equation solution using these known ARE solutions. We need to show that the resulting observer dynamics are stable. For this purpose, it is sufficient (and indeed virtually necessary) to show uniform controllability and observability.

Lemma 6: Consider the system in (1) and suppose it forms the minimal realization (A_i, B_i, C_i) at each operating point $\Lambda_i \in \Lambda_{\rm oper}$. That is $A_i = A(\Lambda_i), B_i = B(\Lambda_i), C_i = C(\Lambda_i)$ and (A_i, B_i) is controllable and (A_i, C_i) is observable $\forall i \in \{1, \ldots, N\}$. Let c be defined as in Section III-B, $\mathcal{A}(t) = A_{c(t)}$ $\forall t$, and $\mathcal{B}(t) = B_{c(t)}$ $\forall t$. Then, $(\mathcal{A}(t), \mathcal{B}(t))$ is uniformly controllable.

Proof: Let $\delta_c > 0$ be defined as in (19) such that $t_{j+1} - t_j \geq \delta_c$ $\forall j \in \mathbb{Z}^+$, and consider an arbitrary time interval of length δ_c, $[s-\delta_c, s)$. We need to show that $\exists \alpha_c, \beta_c > 0$ such that $\alpha_c I \leq M(s-\delta_c, s) \leq \beta_c I$ holds $\forall s \in \mathbb{R}$. It follows from $0 < \delta_c < \infty$ and uniformly boundedness of $\mathcal{A}$ and $\mathcal{B}$ that $\exists \beta_c > 0$ such that $M(s-\delta_c, s) \leq \beta_c I$, $\forall s \in \mathbb{R}$. Since (A_i, B_i) is controllable, if we consider a time interval that does not contain a switching time (i.e., for $s, t, \exists j \in \mathbb{Z}^+$ for which $t_j \leq s < t < t_{j+1}$, with $c(t_j) = i$) and if $(\delta_c/2) \leq t - s \leq \delta_c$, then $\exists \alpha_i > 0$ such that $M(s,t) \geq \alpha_i I$. Let $\tilde{\alpha}_c = \min_i \alpha_i$.

An interval of the form $[s-\delta_c, s)$ will intersect either one interval or two adjacent intervals of the form $[t_j, t_{j+1})$. If one, then we have already established $\tilde{\alpha}_c I \leq M(s-\delta_c, s)$. If two, then taking t_j as the left point on the rightmost interval, we can express the Gramian as

$$\begin{aligned} M(s-\delta_c, s) &= \int_{s-\delta_c}^{s} \Phi(s,t)\mathcal{B}(t)\mathcal{B}'(t)\Phi'(s,t)dt \\ &= \int_{s-\delta_c}^{t_j} \Phi(s,t)B_{c(t_{j-1})}B'_{c(t_{j-1})}\Phi'(s,t)dt \\ &\quad + \int_{t_j}^{s} \Phi(s,t)B_{c(t_j)}B'_{c(t_j)}\Phi'(s,t)dt \\ &= \mathrm{e}^{\left[A_{c(t_j)}(s-t_j)\right]} M(s-\delta_c, t_j)\mathrm{e}^{\left[A'_{c(t_j)}(s-t_j)\right]} \\ &\quad + M(t_j, s). \end{aligned}$$

Noting that either $s - t_j \geq (\delta_c/2)$ or $t_j - (s-\delta_c) \geq (\delta_c/2)$,

$$\alpha_c I \leq M(s-\delta_c, s) \forall s \in \mathbb{R}$$

where

$$\alpha_c = \tilde{\alpha}_c \cdot \min\left\{1, \min_{i, 0 \leq w \leq \frac{\delta_c}{2}} \lambda_{\min}\left[\mathrm{e}^{A_i w}\mathrm{e}^{A_i' w}\right]\right\}$$

and $\lambda_{\min}[\cdot]$ denotes the minimum eigenvalue, it follows that $(\mathcal{A}(t), \mathcal{B}(t))$ is uniformly controllable.

Similarly, the dual property of uniform observability also holds. Because the given system is uniformly controllable and uniformly observable, the dynamics of the observer are stable. Hence, by the Separation Theorem [26], the overall system is stable.

C. Continuously Varying Parameter Analysis

Suppose that the parameter $\Lambda(t)$ is continuously varying. The controller, however, is determined using just the knowledge of the set, say $\Xi_i \in \Xi_{\rm oper}$ at each time in which $\Lambda(t)$ lies. At any instance of time, this set determines a nominal parameter value in $\Lambda_{\rm oper}$, say Λ_i, which is used to set the state-feedback gain part of the controller. The observer gain is however determined from the transient Riccati equation, the solution of which depends on the past values as well as the current value of Λ_i. We earlier established the stability of an interconnection of a controller with a plant with piecewise constant $\Lambda_{c(t)}$, and can appeal to classical results in Lyapunov theory on the robust stability of systems undergoing a perturbation. By rearranging (1) with $\Lambda(t)$ and (7) with $G(\Lambda_{c(t)})$ in (9) and $H(t)$ in (15), the full observer-plant arrangement with $r(t) = 0$ can be written as

$$\begin{aligned} \dot{X}(t) &= \begin{bmatrix} A_{\rm cont}\left(\Lambda_{c(t)}\right) & B_{c(t)}G\left(\Lambda_{c(t)}\right) \\ 0 & A_{c(t)} - H(t)C_{c(t)} \end{bmatrix} X(t) + \Delta(X,t) \\ y(t) &= [C_{c(t)} \quad 0]X(t) \end{aligned} \tag{20}$$

where $e(t) = x(t) - \hat{x}(t)$, $X(t) := [x(t)e(t)]'$, $c(t)$ denotes the index function defined in Section III-B and $A_{\rm cont}(\kappa)$ denotes the control system in (12) and

$$\Delta(X,t) := \begin{bmatrix} \delta A(t) - \delta B(t)G\left(\Lambda_{c(t)}\right) & 0 \\ \delta A(t) - \delta B(t)G\left(\Lambda_{c(t)}\right) - H(t)\delta C(t) & \delta B(t)G\left(\Lambda_{c(t)}\right) \end{bmatrix} \times X(t)$$

reflects the difference between the constant model with $\Lambda_{c(t)}$ and the true plant parameter $\Lambda(t)$ with

$$\begin{aligned} \delta A(t) &:= A(\Lambda(t)) - A_{c(t)} \\ \delta B(t) &:= B(\Lambda(t)) - B_{c(t)} \\ \delta C(t) &:= C(\Lambda(t)) - C_{c(t)}. \end{aligned}$$

In fact, (20) can be classified as a perturbed system, and there exist several standard stability analysis techniques for this class of system [27], [28]. In particular, we borrow a theorem from [27] where the perturbed system is analyzed with an application of Lyapunov's methods.

Theorem 7 ([27]): Consider an auxiliary system

$$\dot{X}(t) = f(X,t) \tag{21}$$

and a perturbed system

$$\dot{X}(t) = f(X,t) + \Delta(X,t). \tag{22}$$

If there exist $\mathfrak{a}, B > 0$ such that solutions of the auxiliary system (21) satisfy

$$\|X(t)\|_2 \leq B\,\|X_0\|_2\,\mathrm{e}^{-\mathfrak{a}(t-t_0)} \quad \forall t \geq t_0 \tag{23}$$

for all[2] initial conditions t_0, X_0, or equivalently, if there exist $c_1, c_2, c_3, c_4 > 0$ and a function $v(X,t)$ such that solutions of the auxiliary system (21) satisfy

$$\begin{aligned} c_1\|X\|_2^2 \le v(X,t) &\le c_2\|X\|_2^2 \\ \dot{v} &\le -c_3\|X\|_2^2 \\ \left\|\frac{\partial v}{\partial X}\right\|_\infty &\le c_4\|X\|_2 \end{aligned} \tag{24}$$

and if there further exists $q \in (0,1)$ such that

$$\|\Delta(X,t)\|_\infty < \frac{(1-q)c_3\|X\|_2}{c_4} \tag{25}$$

another set of positive constants exist such that solutions of (22) satisfy (23) or equally (24).

The results above is now used to prove asymptotic stability of the observer-plant arrangement in (20). Given that the closed-loop of the fixed-point system with a $\Lambda_i \in \Lambda_{\text{oper}}$ is exponentially stable, the appropriate constants can be found to satisfy (23), and equally, there exist appropriate constants and a Lyapunov function $v(X,t)$ that satisfies (23) or (24). Since the system parameter $\Lambda(t)$ is to vary within a bounded subset $\Xi_i \in \Xi_{\text{oper}}$, $\delta A(t)$, $\delta B(t)$, and $\delta C(t)$ are bounded $\forall t$, the perturbation of the system $\Delta(X,t)$ is also bounded. Note that from (17)

$$\sup_t \left\|\Lambda(t) - \Lambda_{c(t)}\right\|_\infty \le \max_i \operatorname{diam}(\Xi_i)$$

where $\operatorname{diam}(S) = \sup_{A,B\in S}\|A-B\|_\infty$. For a fixed bound, one can choose a number of regions, N, large enough to place the associated regions Ξ_i such that the above sup is within that given bound. Given the uniformly continuous dependence of the entries of $\Delta(X,t)$ on the parameter $\Lambda(t)$, it further follows that for a $q \in (0,1)$, a large enough N and associated regions Ξ_i can be chosen such that (25) is satisfied. By choosing a large N, $\Delta(X,t)$ can be restricted to satisfy the condition (25) and hence Theorem 7 provides the asymptotic stability of the closed-loop observer dynamics.

V. Numerical Examples

Let us consider the example in [18]. The system is

$$\begin{aligned} \dot{x}(t) &= A(\Lambda(t))\,x(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}$$

with $x(t_0) = [0\ 0\ 0\ 0\ 0]'$ (for convenience)

$$\begin{aligned} A(\Lambda) &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.1\cos\Lambda - 1 & 1 & 0 \\ 0 & 100 & 0 & 0 & 0 \\ 0 & -100 & 0 & 0 & 1 \\ 0 & 0 & 10\cos\Lambda & 0 & 0 \end{bmatrix} \\ B &= [0\quad 0\quad 0\quad 0\quad 1]', \\ C &= [1\quad 0\quad 0\quad 0\quad 0] \end{aligned} \tag{26}$$

where $-1 \le \Lambda(t) \le 1 \in \mathbb{R}$. To observe clear switchings with less computations $N = 4$, and to satisfy (25), the four *equally spaced* operating points are chosen as $\{-1, -1/3, 1/3, 1\}$ to form Λ_{oper}. In fact, as long as (25) is satisfied, N can be arbitrarily chosen. The design parameters are $\Gamma = 10^{-14}$, $R = 10^{-8}$, and $L = [0.011426, 0.044311, 0.388490, -0.062159, 0.918510]'$ as in [18]. Given the plant time constants $\tau = .01$, the time-varying parameter is chosen with relatively fast variations as $\Lambda(t) = \sin(3t)$ in order to compare the proposed design against that in [18], which is faster than the one used in [18].

[2]A variation in [27] assumes a compact region for X_0, with minor modifications to the theorem criterion.

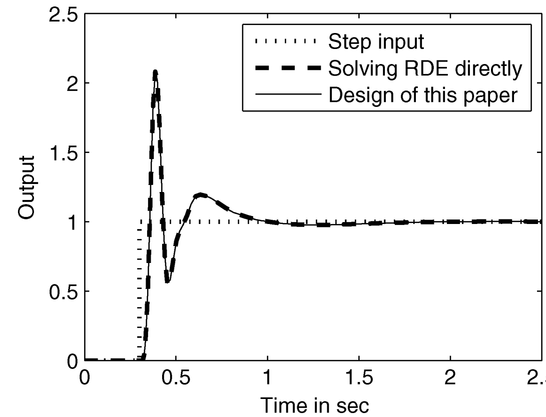

Fig. 1. Step responses for the design in [18] versus our proposed (4 equally spaced operating points) design.

The step responses (a step at $t = 0.3$ s) obtained by solving the RDE directly via [18] with $\Lambda(t)$ and via our proposed design with $\Lambda_{c(t)}$ are shown in Fig. 1. For the first 2.5 seconds both outputs behave indistinguishable with an overshoot just after the step and the exponential convergence as $t \to \infty$, and our new design still satisfies (25). Thus, the stability of the closed loop dynamics is ensured. As N is increased with more equally spaced operating points, more computation is required, but similar performance and stability are observed for the given $\Lambda(t)$. Several other simulations with various mixtures of slow and fast time-varying physical parameters reveal that our proposed design—in comparison to that in [18] which solves the RDE directly—attains similar level of stability and performance.

VI. Conclusion

We have proposed a modification—including computational simplification—of the gain scheduling design of [18]. The design of [18] offers the advantage of permitting fast (though not infinitely fast) parameter variation within the set of approaches to gain-scheduled design. Such an approach may mean that a design cannot be achieved, or may result in restrictive designs. Note that LMI approaches have been suggested to cope with arbitrarily fast time variation, see, e.g., [8], but such schemes may lead to too conservative designs if the variation is not very fast.

The disadvantage of the approach in [18] is that it requires for its implementation the possible solution of asymptotic Riccati equations for an infinite number of different parameter values, and the on-line solution of a RDE with time-varying coefficient matrices. Our modification addresses this computational burden. Only a finite number of AREs is needed to be solved offline, and the RDE solution can be formed using table look-up together with simple matrix operations. Our modification provides, in a sense, an approximation of the design in [18] while similar performance can be achieved with what might in advance be conjectured as quite a rough approximation.

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Consensus of Multi-Agent Networks With Aperiodic Sampled Communication Via Impulsive Algorithms Using Position-Only Measurements

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***Abstract*—In this technical note, an impulsive consensus algorithm is proposed for second-order continuous-time multi-agent networks with switching topology. The communication among agents occurs at sampling instants based on position only measurements. By using the property of stochastic matrices and algebraic graph theory, some sufficient conditions are obtained to ensure the consensus of the controlled multi-agent network if the communication graph has a spanning tree jointly. A numerical example is given to illustrate the effectiveness of the proposed algorithm.**

***Index Terms*—Aperiodic sampled information, consensus, impulsive algorithms, multi-agent networks.**

I. Introduction

It is well known that the consensus problem of multi-agent networks has been widely investigated due to its important applications, including coordinated control of mobile robots, synchronization of dynamical networks, distributed Kalman filtering in sensor networks, load balancing in parallel computers, etc [1]–[4]. Many results have been reported for multi-agent networks with different special features, such as time delay [5], switching topology [6], asynchronous algorithms [4], [7], nonlinear algorithms [8], [9], quantized data [4], noisy communication channel [10], second-order model [11], [12], optimal consensus [13], etc.

Most of the existing works on continuous-time multi-agent networks assume continuous communication among agents. However, in many real-world networks, communication among agents may occur periodically rather than continuously. Therefore, it is more practical to consider continuous-time multi-agent networks with communication at sampling instants. In [16]–[21], consensus problems were addressed for continuous-time multi-agent networks with sampled-data setting. But, all those works assume an equidistant sampling interval, and those results cannot be directly applied to systems whose length of sampling interval is time-varying or uncertain. Consequently, it is desirable to study the consensus problem of multi-agent networks with aperiodic sampling interval. In addition, the existing works often assume that each agent can obtain the information of its full states. However, in some cases, partial states may be unavailable because of technology

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